Examples of Interview test (Basic skills)

Answer the following questions on the differential equation shown in below.

$$\frac{dy}{dt} + f(t)y = g(t)y^2$$

(Q.1) Transform this equation into a linear differential equation by using $w(t) = \frac{1}{y(t)}$. Then, find a general solution $y(t)$ for $f(t) = 0$ and $g(t) = \sin t$.

(Q.2) $f(t) = t$ and $g(t) = e^{\frac{1}{2}t^2} \sin t$, find a general solution $y(t)$.

(Q.3) On (Q.1), the solution of equation satisfies $y(0) = \frac{1}{\alpha}$, where α is a real constant and $\alpha \neq 0$. Let $z = e^{it}$, and calculate the integral $\int_0^{2\pi} y(t)dt$. Here $i = \sqrt{-1}$.

All students in a department of a university have cellular phones and they use either of the two carrier companies: “Itsudemo” and “OhYou!”. It is also found that 100 α % of Itsudemo users change to OhYou! and 100 β % users of OhYou! to Itsudemo every month. Answer the following questions assuming the total number of students is kept constant and 0< α <1, 0< β <1. Also, let $x_1(0)$ and $x_2(0)$ be the shares of Itsudemo and OhYou! users of the present month, respectively.

(Q.1) Let $x_1(1)$, and$x_2(1)$ be the shares of Itsudemo and OhYou! users of the next month, respectively. Find the matrix *A* in the following equation.

$$\begin{bmatrix} x_1(1) \\ x_2(1) \end{bmatrix} = A \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$$

(Q.2) Calculate eigenvalues of the matrix *A*.

(Q.3) Calculate eigenvectors corresponding to the eigenvalues obtained in (Q.2).

(Q.4) Diagonalize the matrix A by using the result of (Q.3).

(Q.5) Find $x_1(n)$ and $x_2(n)$, which are the shares n months later, using the result of (Q.4).

Answer the following questions for the function of t.

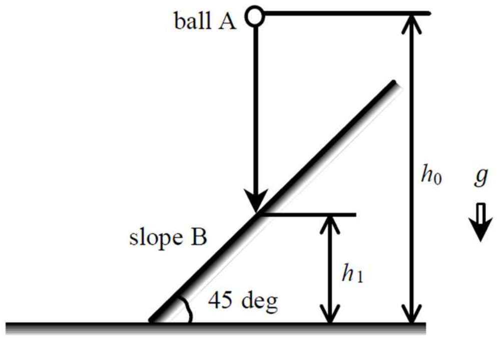
$$I(t) = \int_0^1 \frac{x^t - 1}{\log x} dx$$

where $t \geq 0$ and log is the natural logarithm.

(Q.1) Find $\frac{dI(t)}{dt}$.

(Q.2) Find $I(t)$. Hint：The constant of integration can be obtained by calculating $I(0)$.

Consider the motion of ball A of mass m which can be regarded as a point mass. Let the gravitational acceleration be g.

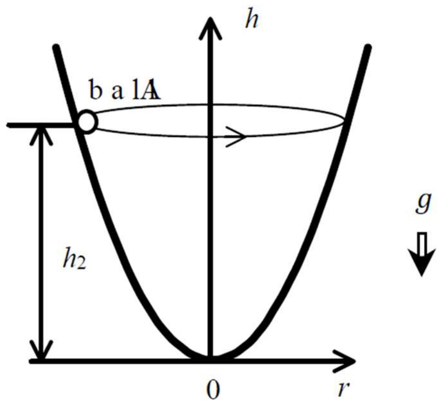


Slope B contacts the horizontal ground at an angle of 45 degrees as shown in this figure. When ball A was dropped from a position at height h0 from the ground with initial velocity 0, it collided with slope B at a position at height h_1. Answer the following questions. Assume that the reflection is specular with a reflection coefficient (coefficient of restitution) of 1.

(Q.1) Calculate the speed of ball A just after collision with slope B.

(O.2) Find the condition for ball A not to collide with slope B again after the first collision with slope B.

(Q.3) Ball A touched the ground after the first collision with slope B. Find the condition of $h_1$for the landing point to be farthest horizontally from the initial position for given h_0, and under this condition, calculate the angle that the velocity of ball A makes with the ground when it touched the ground.

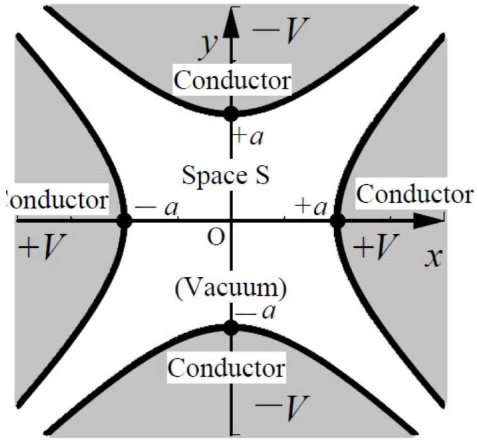


Ball A is moving along the inside surface of a paraboloid of revolution expressed by $h_0 = \frac{1}{a}r^2$ (a is a positive constant) as shown in Figure 2. Answer the following questions. Neglect the friction.

(Q.1) Find the speed of ball A when the ball is circulating while keeping the height at h_2.

As shown in figure as shown in below, let us place four conductors, whose regions are expressed as

$$x^2 - y^2 \geq a^2 \ (-\infty < z < +\infty),$$
$$x^2 - y^2 \leq -a^2 \ (-\infty < z < +\infty).$$



Here x, y, and z denote the Cartesian coordinates. The other region is in vacuum and hereafter referred to as Space S. Electric potentials $+V(>0)$ and $-V$ are applied to the conductors as shown in Figure 1. A charged particle with electric charge $q>0$ and mass m is placed at rest within Space S at time t=0. Answer the following questions.

(Q.1) Prove that the electric potential $\varphi(x, y, z)$ at point (x, y, z) in Space S is given as

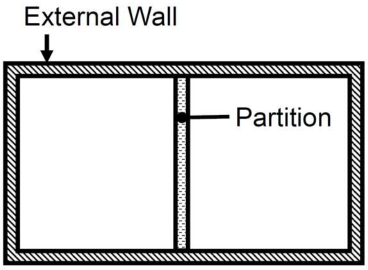
$$\varphi(x, y, z) = \frac{V}{a^2}(x^2 - y^2)$$

Furthermore, find the electric field (E_x, E_y, E_z) at point (x, y, z) in Space S.

(Q.2) Express the equation of motion for the charged particle in Space S in x, y, and z directions.

(Q.3) Find a possible range of initial position of the charged particle for which the solution of the equation of motion in (Q.2) represents a periodic motion within Space S. Furthermore, find the period of this motion.

Consider a tank surrounded by insulated external walls and divided into two chambers by a partition, as shown in below. Each chamber is charged with an ideal gas. Answer the following questions about this system.



(Q.1) The partition is an insulated wall and fixed at the center of the tank. The right chamber is evacuated. The left chamber is charged with the ideal gas A with a pressure of P_{A0}, a temperature of T_{A0} and a mass of m. The specific heat at constant volume per mass is C_{VA} and the ratio of specific heat is γ. At a certain instant, the partition is removed instantaneously, and the gas A is expanded adiabatically and freely, and then the state reaches equilibrium. Express the temperature in the equilibrium state and the entropy variation by this process.

Next, the partition is fixed at a certain position and the left and right chambers are filled with the gas A and gas B, respectively. The temperatures of those gases in the initial condition are T_{A0} and T_{B0}, the pressures are P_{A0} and P_{B0}, and the specific heats at constant volume per mass are C_{VA} and C_{VB}, respectively. Then, the charged mass and the ratios of specific heat of both gases are the same as m and γ respectively. Answer the following questions.

(Q.2) Express the volume ratio of the gases A and B (V_{A0}/V_{B0}), in the initial condition.